

The Markowitz problem: updated staging, analytical problem solving, borders and possible areas of use of analytical method

*V.R. Kigel, candidate of economic Sciences, associate Professor,
Professor of the chair of mathematical methods and statistics
University of Economics and law "KROK"
(Kigel@ukr.net)*

Research methodology. Methods of economic-mathematical modelling and operations research helps the Markowitz in the updated production, justified method of its analytical solution, outlines the limits and shows the potential of the sphere of securities market and the Forex market use it.

Results. Task Markowitz in the updated formulation can be written as:

$$\hat{z} \doteq \sum_{j=1}^n \bar{d}_j x_j + k \sqrt{\sum_{i=1}^n \sum_{j=1}^n \rho_j \sigma_i \sigma_j x_i x_j} \rightarrow \max, \quad \sum_{j=1}^n x_j = I, \quad x_j \geq 0, \quad j = \overline{1, n},$$

where x_i, K, x_n the volume of investment in each of the areas of investment, \hat{z} - deterministic equivalent of random total income financial portfolio, calculated according to the individual preferences of the investor. There is the solution of this task for each of the main types of the attitude of the investor's risk neutrality ($k = 0$), the propensity ($k > 0$) or reluctance ($k < 0$). In particular, for the case predisposed to risk reasonably such results:

$$\sigma_z = \frac{-kI}{\sqrt{\gamma^2 - \beta(\alpha - k^2)}},$$

$$t = \frac{I}{\sqrt{\gamma^2 - \beta(\alpha - k^2)}} R^{-1} \left(a^T + \frac{-\gamma + \sqrt{\gamma^2 - \beta(\alpha - k^2)}}{\beta} b^T \right),$$

$$x_j = \frac{t_j}{\sigma_j}, \quad j = \overline{1, n}, \quad \text{where } \alpha = aR^{-1}a^T, \beta = bR^{-1}b^T, \gamma = aR^{-1}b^T,$$

$$a_j = \frac{\bar{d}_j}{\sigma_j}, \quad b_j = \frac{1}{\sigma_j}, \quad j = \overline{1, n}.$$

Novelty. Results presented in the article to author's knowledge are new.

Practical significance. The method is illustrated with specific examples possibility of use of instruments for the management of a portfolio of financial assets and foreign currency reserve.